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Determination of Orthometric Heights of Points Using Gravimetric/GPS and Geodetic Levelling Approaches

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ABSTRACT

The importance of practical, as well as orthometric heights in engineering cannot be underestimated as it is required for the determination of proposed construction levels and to direct the flow of water. This study presents the determination of orthometric heights of points using gravimetric/GPS and geodetic levelling approaches and compares the resolution of the two approaches to determine which of the methods is better for orthometric height determination in the study area. A total of 59 stations were occupy for gravity observation using Lacoste and Romberg (G-512 series) gravimeter to obtain the absolute gravity values of the points. GNSS observation was carried out in static mode using South GNSS receivers to obtain the positions and ellipsoidal heights of the points. The modified Stokes' integral was applied to obtain the geoid heights of the points. Similarly, levelling was carried out using the geodetic level to obtain the level heights of the points. The orthometric correction was applied to the geodetic levelling results to obtain precise level heights of the points. The RMSE index was applied to compute the accuracy of the geoid models. The computed result shows that orthometric heights can be obtained in the study area using the two models with an accuracy of 0.3536m. Z-test was carried out to determine if there is any significant difference between the two methods. The test results show that statistically, there is no significant difference between the two methods. Hence, the two methods can be applied for orthometric heights determination in the study area.

Key words: Orthometric Heights, Gravimetric, GPS, Geodetic, Levelling

1. INTRODUCTION

Height is one of the important components needed to determine the position of any required point on or below the earth's surface. Different height systems have been adapted depending on the reference surface and the method of its determination. Among these height systems, are dynamic heights, orthometric heights, normal heights, and geodetic heights. Orthometric heights, mostly used are been referred to mean sea level are very important practically because of their geocentric and physical significance in engineering construction. Orthometric heights are normally obtained from spirit levelling

and gravity measurement (Moka, 2011; Tata and Ono, 2018). Orthometric height determination has a significant role in geodesy, and it has wide-ranging applications in various fields and activities. Orthometric height is the height above or below the geoid along the gravity plumb-line (Peprah and Kumi, 2017; Tata and Ono, 2018). It is the distance, measured positive outwards or negative inwards along the plumb-line, from the geoid (zero orthometric height) to a point of interest, usually on the topographic surface.

The necessity for a refined geoid models has been driven mainly by the demands of users of the Global Positioning System (GPS), who must convert GPS-derived ellipsoidal heights to orthometric heights (Opaluwa and Adejare 2011) to make them compatible with the existing orthometric heights on the vertical datum. GPS and orthometric height data are commonly used to verify gravimetric geoid models on land, and thus indirectly the data, techniques, and theories are utilized (Engelis *et al.*, 1984; Sideris *et al.*, 1992; Featherstone, 2014).

Orthometric height (H) of a point P on the surface of the earth is its distance from the geoid, P_0, measured along the plumb-line normal to the geoid as given in Figure 1. It is the vertical separation between the geopotential passing through the point, P on the earth's surface and the geoid (the reference equipotential surface). Since equipotential surfaces are not parallel, this plumb-line is a bend line. Orthometric heights can be determined using geometric or trigonometric levelling (Odumosu *et al.*, 2018). This can be obtained as:

$$H = \frac{c}{\bar{g}} \tag{1}$$

Mathematically, Orthometric height is the ratio of geopotential number (C) to mean gravity value ($\bar{g}$) along the plumb-line between the geoid and the point, P on the earth surface given as (Heiskanen and Moritz 1967)

$$H = \frac{1}{H}\int_{o}^{H} g(z)dz \tag{2}$$

Where g (z) is the actual gravity at the variable point, P of the height Z as given in Fig. 1.

$$\bar{g} = g + 0.424H \tag{3}$$

Where gravity is observed at the surface point, P in gals and H is its height in kilometres.

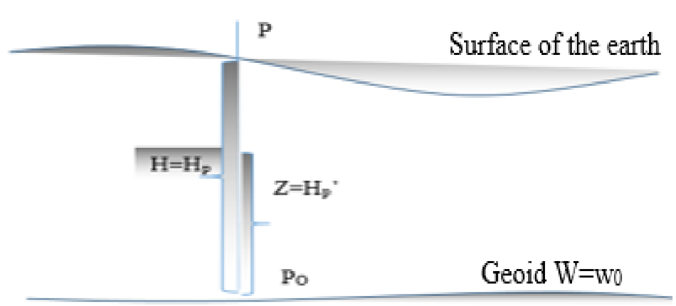

Fig.1: The Prey Reduction

Applying equation (3) in equation (1) gives what it refers to as Helmert Orthometric height as given in equation (4).

$$H = \frac{c}{g + 0.424H} \tag{4}$$

In equations (2), (3) and (4), H is the Orthometric height of the point, P but because $\bar{g}$ does not depend strongly on H, the uncorrected height of the point can be used in equations (3) and (4) for practical purposes. Following Heiskanen and Moritz (1967), $\overline{g}$ can be computed to a sufficient accuracy as

$$\bar{g} = \frac{1}{2}(g - g^{o}) \tag{5}$$

Where g is the gravity measured at the surface point, g^{o} is the gravity value computed at the corresponding point, P_0 on the geoid by prey reduction as given in Figure 1. Prey reduction is performed according to the remove-compute-restore (R-C-R) procedure (Moka, 2011). Gravity at P_0 (geoid) is thus given by $g^{\circ} = g + 0.0848H_{\rho}$. Practically, the orthometric height difference is obtained from measured height difference by adding Orthometric correction to it. For two points A and B connected by levelling, we have

$$\Delta H_{AB} = \Delta n_{AB} + OC_{AB} \tag{6}$$

Where, ΔH_{AB} is Orthometric height difference between points A and B, Δn_{AB} is levelled height difference between the two points, A and B and OC_{AB} is Orthometric correction between the points and it is computed as

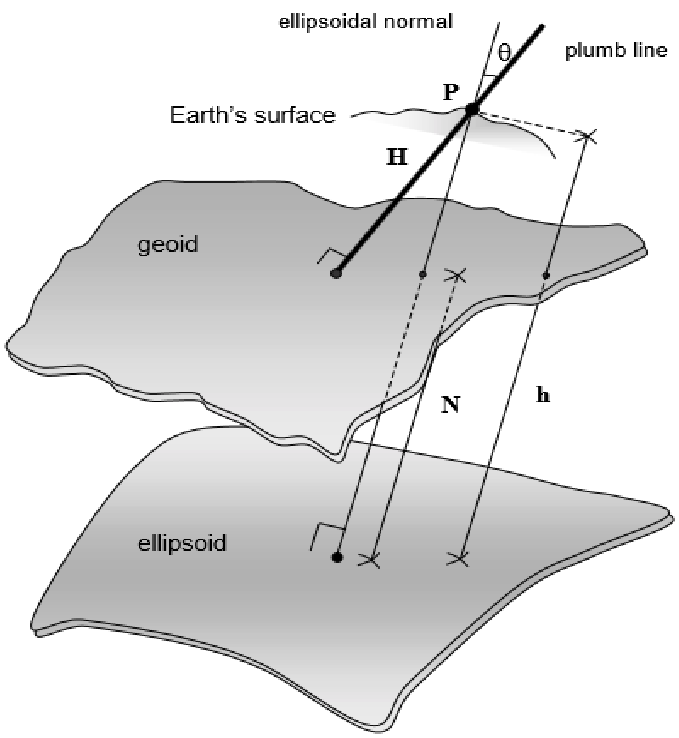

Fig. 2. Relationship between Ellipsoidal, Geoid and Orthometric Heights (Fotopoulos 2003, Herbert and Olatunji 2021)

$$OC_{AB} = \sum_{A}^{B} \frac{g - \gamma^{o}}{\gamma^{o}} \delta n + \frac{\bar{g}_A - \gamma^{o}}{\gamma^{o}} H_A - \frac{\bar{g}_B - \gamma^{o}}{\gamma^{o}} H_{AB} \tag{7}$$

Where g is the gravity values of each section, $\bar{g}_A$ is the mean value of gravity along the plumb-line of A, $\bar{g}_B$ is the mean value of gravity along the plumb-line of B, H_A is the height of A, H_B is the height of B, γ_o is an arbitrary constant normal gravity, say 45° Latitude, and δ_n is levelling increment for each set-up.

The optimal combination of geometric heights obtained from Global Positioning System (GPS) measurements with geoidal undulations derived from a gravimetric geoid model, to determine orthometric heights relative to a vertical geodetic datum, is well suited for many practical applications as given in (Fig. 2) and equations (8) and (9a). This process, referred to as GPS/levelling geoid is based on a simple geometrical relationship that exists between the geodetic surfaces given by Heiskanen and Moritz, (1967).

$$H = h - N \tag{8}$$

Gravimetric Approach

The word 'gravimetric' originates from gravity, which can be defined as the resultant effect of gravitation and centrifugal forces of rotating Earth (Heiskanen and Moritz, 1967; Fubara, 2007). The gravimetric geoid is the oldest method of geoid determination (Fubara, 2007). The principle of this method requires that the entire earth's surface be sufficiently and densely covered with gravity observations. Practically, a dense gravity net around the computation n point and reasonably uniform distribution of gravity measurement outside are sufficient. Then, gravity approximation is inevitable, to fill the gap with extrapolated values (Featherstone *et al.* 1998). Depending on the area of coverage, gravimetric geoid may be global, regional, or local. Regional gravimetric geoid models are the best because they are of high resolution, local gravity and terrain data are often added to the global geopotential model and optimized for the area of interest (Featherstone *et al.* 1998). However, the application of this technique is mainly dependent on the availability of high-resolution gravity data (Tata and Ono, 2018). The original technique is based on Stoke's-Integral equation (9) and the use of accurately obtained absolute gravity data (Heiskanen and Moritz, 1967).

The Geoidal Undulation (N) at any point P (φ, λ) on the Earth's surface can be computed using the evaluation of the Stokes' Integral, given by Bernhard and Moritz (2005) as

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g s(\psi) \delta\sigma \tag{9}$$

Where N is the geoidal undulation obtained from a gravimetric geoid, γ is gravity anomaly, $\iint_{\sigma}$ an integral extended over the whole Earth, R is the mean radius of the Earth, Δg is the gravity anomaly known everywhere on the Earth, S(ψ) is Stokes' function between the computation and integration points, given as:

$$S(\psi) = \csc\left(\frac{\psi}{2}\right) - 6\sin\left(\frac{\psi}{2}\right) + 1 - 5\cos\psi - 3\cos\psi \ln\left\{\sin\left(\frac{\psi}{2}\right) + \sin^2\left(\frac{\psi}{2}\right)\right\} \tag{10}$$

While the surface spherical radius, ψ_o is computed as given by Oduyebo *et al.* (2019) as

$$\cos\psi = \sin\varphi \sin\varphi' + \cos\varphi \cos\varphi' \cos(\lambda' - \lambda) \tag{10b}$$

Where φ the mean latitude of the points is, φ' is the latitude of individual point, λ is the mean longitude of the points, λ' is the longitude of individual point and (dσ) is the differential area on the geoid. Using the integration of the modified Stokes' integral given in equation (11), the geoidal undulations of points can be computed if their gravity anomalies, the normal gravity, and geographical positions are known.

$$N = \frac{r\Delta g}{8\gamma}\begin{pmatrix} -6\sin^2\psi_o \ln\left\{\sin\left(\frac{\psi_o}{2}\right) + \sin^2\left(\frac{\psi_o}{2}\right)\right\} + 16\sin\left(\frac{\psi_o}{2}\right) + 12\sin^2\left(\frac{\psi_o}{2}\right) \\ -24\sin^3\left(\frac{\psi_o}{2}\right) - 12\sin^4\left(\frac{\psi_o}{2}\right) - 4\cos\psi_o + 5\cos 2\psi_o - 1 \end{pmatrix} \tag{11}$$

Where N is the geoidal height of individual point, ψ_0 is the surface spherical radius as computed using equation (10b), γ is the theoretical or normal gravity, Δg is the gravity anomaly, and $r = R$ is the mean radius of the earth.

Stokes' formula, equation (9), often described as a conventional solution of the geodetic boundary value problem. It computes absolute geoid and requires that gravity values are all over the Earth to compute geoidal undulations. This makes its application to be expensive, tedious, and time-consuming. Hence, there is a need to develop a computational tool that will be user friendly, economical, and fast in computation.

Gravity Anomaly and Normal Gravity

The gravity anomaly (Δg) which is the major input in geoid computation is the difference between the magnitudes of the reduced absolute gravity (g) at a point on the geoid, and the normal gravity (γ) on the reference ellipsoid (Δg = g- γ). The normal gravity is the theoretical gravity value of a point computed on a specified ellipsoid. It is latitude dependent component. The Somigliana's formula for the computation of the normal, as well as the theoretical gravity of points on a specified ellipsoid, is:

$$\gamma = \frac{{}^{a}\gamma_a \cos^2\varphi + {}^{b}\gamma_b \sin^2\varphi}{\sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}} \qquad (12)$$

Where a, and b are respectively the semi-major and semi-minor axes of the ellipsoid, and γ_a, and γ_b, are normal gravity at the equator and the pole of the ellipsoid, respectively. The gravity anomaly has traditionally been adopted as the boundary value to model the disturbing potential, and ultimately the geoid undulation, which is Stokes' integral.

Geometric Approach (GPS/Geodetic Levelling)

The method of GPS/geodetic levelling for obtaining geoidal heights cannot be assumed as a new theory. In fact, as a result of case studies that have been conducted by different researches, (Essam 2014; Aleem, 2014; Eteje *et al.*, 2018) it is evidenced that the GPS/geodetic levelling can provide a possible alternative to traditional techniques of levelling measurement, which is tedious, time-consuming and prone to errors over a long distance.

Orthometric heights determination from a general perspective is directly dependent on the gravity field. Geometric levelling is the conventional approach used in the determination of orthometric height which is known to be time-consuming, prone to human error, and cumbersome, especially in large areas, very rough terrain, and over a long distance. Apart from the difficulty faced during field measurement, a lot of time and energy is spent during data reduction and adjustment thereby making it highly capital exhaustive to establish a countrywide high-resolution levelling network. Furthermore, the availability of this data in the study area is inadequate. The absence of gravity data to determine geoidal heights has made it difficult, among other problems, to determine orthometric heights which have necessitated the adoption of different height systems that are unharmonious to one another. Thus, most geodetic and engineering applications are either referenced to the ellipsoid or other arbitrary height systems and all of these do not represent the definite form (geoid) of the earth over the study area. Hence the need for the determination of orthometric heights of points using geometric and gravimetric approaches to compare the resolution of the two approaches for the best fitting orthometric height for the study area.

Z-Test Statistics

A Z-test is carried out to dictate if two samples means are statistically different from each other. This is done by comparing the means and variances of both samples. The two hypothesis tests that are normally carried out are the null hypothesis (Ho) and the alternative hypothesis (Hi).

For Ho: $\mu_1 = \mu_2$: there is no significant difference between the means of populations 1 and 2.
For Hi: $\mu_1 \neq \mu_2$: there is a significant difference between the means of populations 1 and 2.

The model for Z-test computation is given as:

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_P^2 (1/n_1 + 1/n_2)}} \quad (13)$$

Where μ is the population mean, $\bar{X}$ is the sample mean, n is the sample size, and σ is the sample standard deviation.

2. METHODOLOGY

In this section, the gravimetric method of geoid determination was adopted using the Stokes' integral, which is the basis of the gravimetric method of geoid determination, as well as orthometric height computation. Geodetic levelling observation was carried out in loops and reduced using the height of instrument method. The obtained heights were corrected for orthometric correction using equation (7) since gravity values were observed in this study. A total of fifty-nine (59) existing GPS stations within the study area were used as common points for the two approaches during the observation. The gravity measurements were carried out by professionals from the National Geological Survey Agency (NGSA), Nigeria. A Lacoste and Romberg (G-512 series) gravimeter, was used to measure the gravity values of all the stations. South GNSS receivers were used in static mode for the determination of the positions and the ellipsoidal heights of each of the points while a geodetic level was used to determine the level heights of the points. The normal gravity values of the points were computed using equation (12). The gravity anomalies of the points were computed by finding the differences between the observed absolute gravity values of the points and their respective normal gravity values, as detailed in section 1.2.1. The common corrections needed in a gravity survey such as corrections for latitude, drift, tide, free air, and Bouguer were all applied accordingly for a better result. The gravimetric geoid heights were computed using the modified Stokes' integral, as well as equation (9). In the computation of the gravimetric geoid heights, the entire study area was subdivided into different compartments of grid lines of 1'x1' (1800 x 1800) m². The subdivision was achieved by grid lines of a fixed coordinate system (φ, λ) forming square blocks. Hence the computed, as well as the gravimetric orthometric heights of the fifty-nine (59) stations were obtained by computing the differences between the ellipsoidal heights (h) and the computed gravimetric geoidal heights (N). A Z-test, as well as hypothesis testing, was carried out to determine whether there was a significant difference between the means of the gravimetric orthometric heights and the geodetic orthometric heights. The Z-test computation was carried out using equation (13), as well as a Microsoft Excel 2013 program. The accuracy of the two methods geoid models was computed using the RMSE index. The computation method is detailed in Eteje and Oduyebo (2018) and Eteje *et al.* (2019). Fig. 3 shows the schematic methodology adopted in this study.

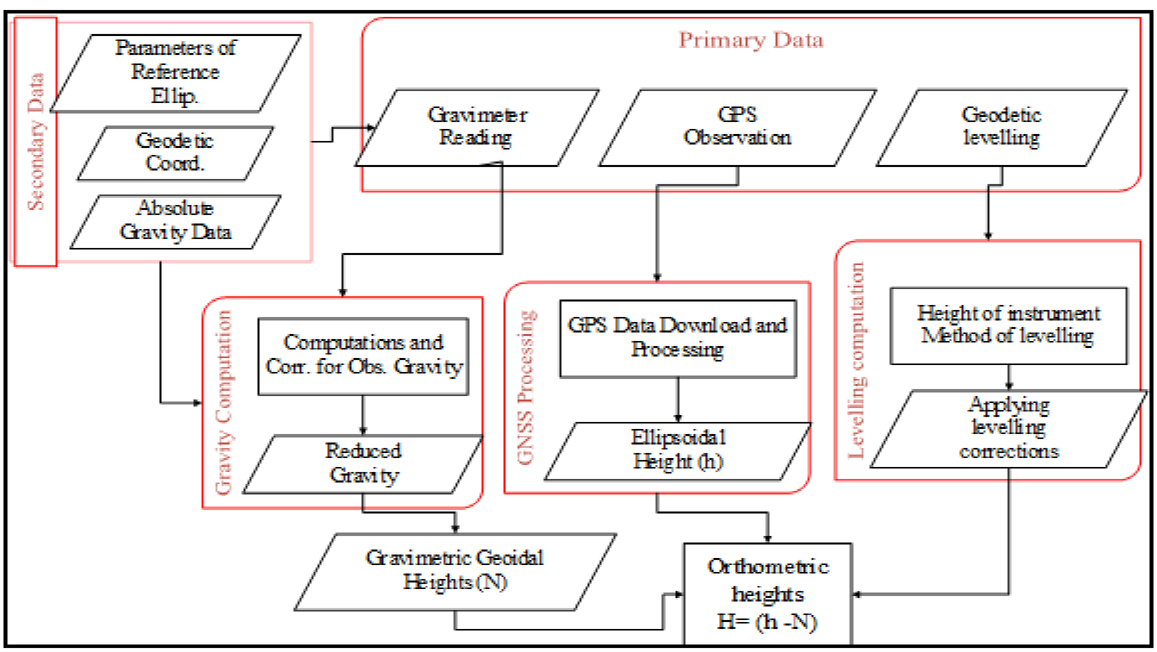

Fig. 3. Research Methodology Flow Chart

3. RESULTS AND ANALYSIS

The results presented in Table 1 are the geographical coordinates of the existing GPS control points within the study area, ellipsoidal heights obtained from GNSS observation, gravimetric geoid heights determined via Stokes' integral, gravimetric orthometric heights computed from the differences between the gravimetric geoidal heights and the ellipsoidal heights, and orthometric heights obtained via geodetic levelling. Fig. 3 to 5 are respectively the contour and 3D plots of the ellipsoidal heights, gravimetric and orthometric height.

Table 1. Ellipsoidal Heights, Gravimetric Geoidal Heights, Gravimetric Orthometric Heights, and Geodetic Orthometric

Stations	LAT. [o]	LONG. [o]	Ellipsoidal Height (h) [m]	Gravimetric Geoid Height (N) [m]	Gravimetric Orthometric Height (H) [m]	Geodetic Levelling Orthometric Height (H)[m]
GPSA72S	7.270799	5.167048	359.9130	13.4430	346.4700	346.4700
GPSA73S	7.270721	5.165108	358.6052	13.6513	344.9539	345.1465
GPSA75S	7.271863	5.162073	352.0484	13.9463	338.1021	338.3880
GPSA76S	7.272547	5.159758	350.3126	13.7726	336.5400	336.6660
GPSA77S	7.273701	5.156847	348.2843	14.1975	334.0868	334.6510
GPSA78S	7.273667	5.155197	351.0061	14.0918	336.9143	337.3650
GPSA79S	7.274355	5.152743	356.3205	13.5685	342.7520	342.5380
GPSA80S	7.275406	5.150703	359.4789	14.1616	345.3173	345.8310
FG28	7.275991	5.148142	359.5066	13.2755	346.2311	345.8170
FG29	7.276984	5.146602	353.7181	13.4880	340.2301	340.1150
GPSA81S	7.272334	5.160786	351.9264	13.8976	338.0288	338.2150
GPSA82S	7.270241	5.160641	347.8124	13.2327	334.5797	334.1020
GPSA83S	7.261542	5.162506	363.9258	14.1605	349.7653	349.7650
GPSA84S	7.258587	5.162796	360.0313	13.7050	346.3263	345.8400
GPSA85S	7.256818	5.163624	353.4029	13.5549	339.8480	339.1970
GPSA45S	7.244252	5.191632	347.4330	14.5513	332.8817	332.6211
GPSA46S	7.247510	5.192651	346.7350	14.2326	332.5024	331.9146
GPSA25S	7.240000	5.196047	346.6750	13.7013	332.9737	332.4130
GPSA27S	7.234097	5.198924	355.3490	12.6683	342.6807	341.1599
GPSA29S	7.234391	5.198202	356.0379	12.6145	343.4234	342.6439
GPSA30S	7.232633	5.198040	358.8226	12.6078	346.2148	345.0194
GPSA31S	7.230726	5.198220	363.7909	13.2375	350.5534	350.0856
GPSA33S	7.224348	5.198436	361.8296	13.3533	348.4763	347.6230
GPSA35S	7.218419	5.197596	354.2590	13.5397	340.7193	340.0960
		MEAN	**348.1680**	**13.2471**	**334.9209**	**334.7821**
		STDV	**10.5637**	**0.6653418**	**10.204114**	**10.218727**

Fig. 3a is the contour plot of the ellipsoidal heights obtained from the data set in Table 1. The plotting was done with Surfer software using the kriging gridding method at 1.5m grid interval to present graphically, the shape of the obtained ellipsoidal heights. From (Fig. 3a), it can be seen that the minimum and the maximum contour values are respectively 300m and 365m which implies that ellipsoidal heights can be obtained in the study area within the range of 300m to 365m. (Fig. 3b) is the surface plot of the ellipsoidal heights. It was also done to present graphically, the shape of the ellipsoidal heights of the study area. It can be seen from (Fig. 3b) that the lowest and the highest values of the ellipsoidal heights are respectively 305m and 360m as indicated by the colour scale bar. This also implies that ellipsoidal heights can be obtained in the study area within the range of 305m to 360m using the 3D surface.

Fig. 4a and 4b are respectively contour and 3D surface plots of the orthometric heights obtained via the gravimetric approach given in Table 1. These were done to present graphically, the shape of the gravimetric orthometric heights. It can be seen from Figure 4a that the minimum and the maximum contour values at 1.5m grid interval are respectively 290m and 350m. This implies that, with the gravimetric geoid, orthometric heights can be interpolated in the study area within the range of 290m to 350m using the contour map. Also from (Fig. 4b), it can be seen as indicated by the colour scale bar that the lowest and the highest orthometric heights are respectively 290m and 350m. This implies that the gravimetric orthometric heights of the study area range from 290m to 350m.

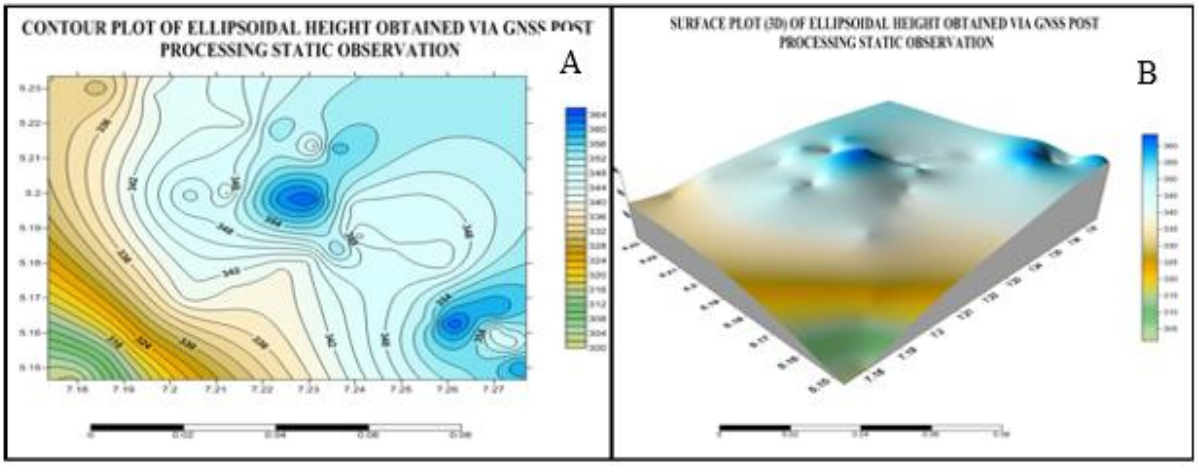

Fig. 3. Contour and 3D Surface plots of Ellipsoidal Heights Obtained from GNSS

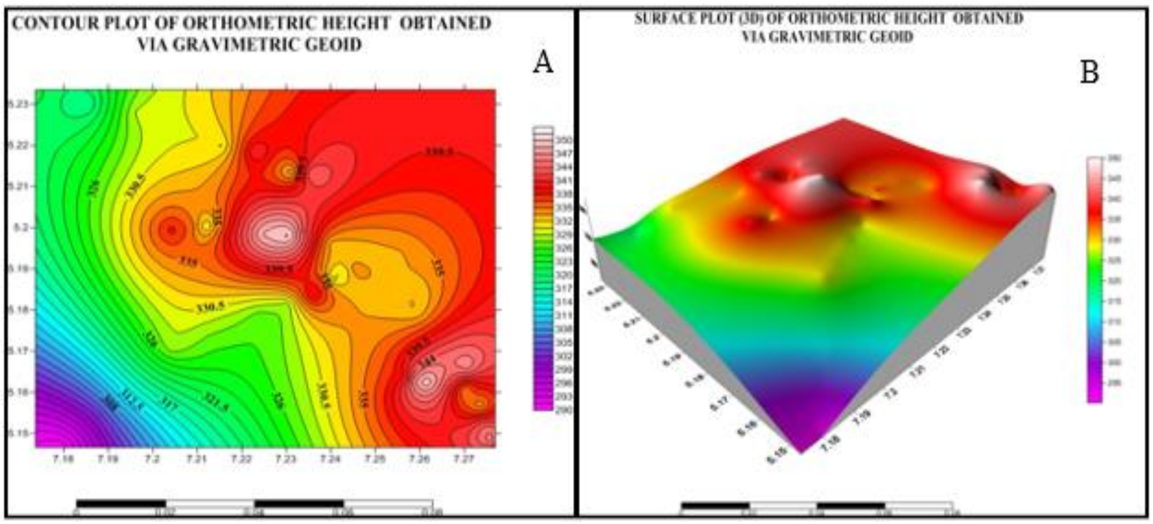

Fig.4. Contour and 3D Surface Plots of Orthometric Height via Gravimetric Geoid

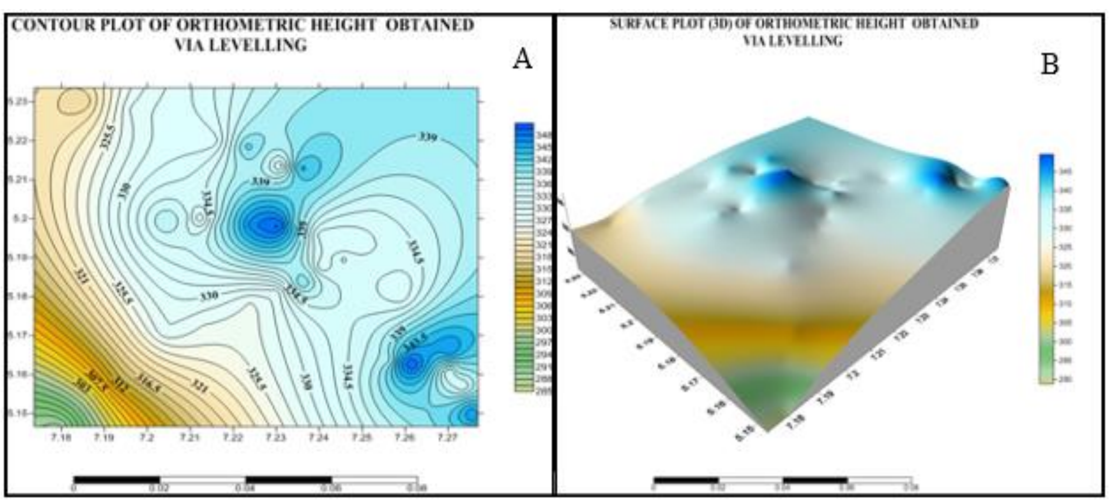

Fig. 5. Contour and 3D Surface Plots of Orthometric Height via Geodetic Levelling

Fig. 5a and 5b are respectively contour and 3D surface plots of the orthometric heights obtained from the geodetic levelling given in Table 1. These were also done to present graphically, the shape of the geodetic levelling orthometric heights. It can also be

seen from (Fig. 5a and 5b) (as indicated by the colour scale bar) that the minimum and the maximum contour values at 1.5m grid interval are respectively 300m and 356m. This shows that orthometric heights can be obtained in the study area within the range of 300m to 356m with the geodetic levelling method.

Table 2 presents the Z-test results. This was done to determine if there was statistically, a significant difference between the means of the gravimetric and the geodetic orthometric heights. The hypothesis testing was performed by comparing the means of the gravimetric and the geodetic levelling orthometric heights. This is a two-tail test where the null hypothesis is accepted if the calculated Z value is smaller than the Z critical. Also, the null hypothesis is accepted if the P-value is greater than the significant level of α at 0.05 statistic, is more than the upper limit and less than the lower limit of the table statistic. It can be seen from Table 2 that the computed P-value is 0.95 which is greater than the significance level of α at 0.05 and the computed Z is 0.056 which is smaller than the z critical at 1.96. Since $P > 0.05$ and Z-calculated < Z-critical, the null hypothesis (Ho) was accepted. This shows that there is no statistically significant difference between the mean of gravimetric orthometric heights and that of the geodetic levelling orthometric heights. This, in turn, implies that there is no statistically significant difference between the results of the two methods. Hence the two methods can be applied interchangeably in the study area.

Table 2. Z- Test Analysis

Z-Test: Two-Sample for Means		
	Gravimetric Orthometric Height (H) [m]	**geodetic** *Levelling Orthometric Height (H) [m]*
Mean	334.5032742	334.3930126
Known Variance	112.619	111.554
Observations	59	59
Hypothesized Mean	0	
z	0.056566457	
P(Z<=z) one-tail	0.477445278	
z Critical one-tail	1.644853627	
P(Z<=z) two-tail	0.954890555	
z Critical two-tail	1.959963985	

Table 3 shows the computed Root Mean Square Error (RMSE) of the two methods models of the study area. It was done to present the accuracy of the two models of the study area. The accuracy, as well as the RMSE of the models, was computed as shown in Table 3. It can be seen in Table 3 that the RMSE of the two orthometric height models of the study area is 0.3536m. It means that orthometric heights can be obtained in the study area with an accuracy of 0.3536m with the two models.

Table 3: Computation of RMSE/Accuracy of the Two Methods Models

Stations	**Gravimetric Orthometric Height (H) [m]**	**Geodetic Levelling Orthometric Height (H)[m]**	**Diff. b/w Gravimetric & Geodetic Orthometric Heights**	**Square of the Diff. b/w Gravimetric & Geodetic Orthometric Heights**
GPSA72S	346.4700	346.4700	0.0000	0.0000
GPSA73S	344.9539	345.1465	-0.1926	0.0371
GPSA75S	338.1021	338.3880	-0.2859	0.0817
GPSA76S	336.5400	336.6660	-0.1260	0.0159
GPSA77S	334.0868	334.6510	-0.5642	0.3183
GPSA78S	336.9143	337.3650	-0.4507	0.2031
GPSA79S	342.7520	342.5380	0.2140	0.0458
GPSA80S	345.3173	345.8310	-0.5137	0.2639
FG28	346.2311	345.8170	0.4141	0.1715
FG29	340.2301	340.1150	0.1151	0.0132
GPSA81S	338.0288	338.2150	-0.1862	0.0347
GPSA82S	334.5797	334.1020	0.4777	0.2282

GPSA83S	349.7653	349.7650	0.0003	0.0000
GPSA84S	346.3263	345.8400	0.4863	0.2365
GPSA85S	339.8480	339.1970	0.6510	0.4238
GPSA45S	332.8817	332.6211	0.2606	0.0679
GPSA46S	332.5024	331.9146	0.5878	0.3455
GPSA25S	332.9737	332.4130	0.5607	0.3144
GPSA27S	342.6807	341.1599	1.5208	2.3128
GPSA29S	343.4234	342.6439	0.7795	0.6076
GPSA30S	346.2148	345.0194	1.1954	1.4290
GPSA31S	350.5534	350.0856	0.4678	0.2188
GPSA33S	348.4763	347.6230	0.8533	0.7281
GPSA35S	340.7193	340.0960	0.6233	0.3885
Sum of Square of the Diff. b/w Gravimetric & Geodetic Orthometric Heights (A)				**0.3536**
Square Root of (A) = RMSE/Accuracy				**0.5946**

4. CONCLUSION

The study has shown the potentials on the use of ellipsoidal heights, geoidal heights, and geodetic levelling data for precise orthometric heights determination. It has also demonstrated the possibility of using gravimetric observations combined with GNSS data or geometric approach for orthometric height computation to ascertain the best fitting approach for the study area. The results obtained from this study, as well as the two approaches, show that there is no difference between the two methods as indicated by the Z-test results. The RMSE computation results also show that orthometric heights can be obtained in the study area using any of the two approaches with an accuracy of 0.3536m. Thus, the two methods can be applied interchangeably in the study area for orthometric heights determination.

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Conflict of Interest:

The authors declare that there are no conflicts of interests.

Data and materials availability

All data associated with this study are present in the paper.

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